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## Solution of the linear and non-linear differential equations by using Homotopy perturbation method

## KEYWORDS

System of linear ordinary differential equations; Abelian differential equations; Homotopy perturbation method; Numerical simulation.

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**ABSTRACT** In this paper we use He's Homotopy perturbation method is applied to solve a system of linear ordinary differential equations of the first order and some first order non-linear ordinary differential equations like Abelian differential equations. The method yields solutions in convergent series form with easily computable terms. The result shows that these methods are very convenient and can be applied to a large class of problems. Some numerical examples are given to the effectiveness of the method. Our analytical results are compared with the numerical results and a satisfactory agreement is noted.

## INTRODUCTION

A system of ordinary differential equations of the first order can be considered as [1-4]:

$$\begin{aligned} y_1' &= f_1(x, y_1, \dots, y_n) \\ y_2' &= f_2(x, y_1, \dots, y_n) \\ &\vdots \\ y_n' &= f_n(x, y_1, \dots, y_n) \end{aligned} \quad (1)$$

where each equation represents the first derivative of one of the unknown functions as a mapping depending on the independent variable  $x$ , and  $n$  unknown functions  $f_1, f_2, \dots, f_n$ . Since every ordinary differential equation of order  $n$  can be written as a system consisting of  $n$  ordinary differential equation of order one, we restrict our study to a system of differential equations of the first order.

Linear and non-linear phenomena are of fundamental importance in various fields of science and engineering. Most models of real-life problems are still very difficult to solve. Therefore, approximate analytical solutions such as Homotopy perturbation method (HPM) [5-16] were introduced. This method is the most effective and convenient ones for both linear and non-linear equations. Perturbation method is based on assuming a small parameter. The majority of non-linear problems, especially those having strong non-linearity, have no small parameters at all and the approximate solutions obtained by the perturbation methods, in most cases, are valid only for small values of the small parameter. Generally, the perturbation solutions are uniformly valid as long as a scientific system parameter is small. However, we cannot rely fully on the approximations, because there is no criterion on which the small parameter should exist. Thus, it is essential to check the validity of the approximations numerically and/or experimentally. To overcome these difficulties, HPM have been proposed recently.

Recently, many authors have applied the Homotopy perturbation method (HPM) to solve the non-linear boundary value problem in physics and

engineering sciences [5-8]. Recently this method is also used to solve some of the non-linear problem in physical sciences [9-11]. This method is a combination of Homotopy in topology and classic perturbation techniques. Ji-Huan He used to solve the Light hill equation [8], the Diffusion equation [9] and the Blasius equation [10-11]. The HPM is unique in its applicability, accuracy and efficiency. The HPM uses the imbedding parameter  $p$  as a small parameter, and only a few iterations are needed to search for an asymptotic solution.

## 2. Basic concepts of the Homotopy perturbation method [5-16]

To explain this method, let us consider the following function:

$$D_\alpha(u) - f(r) = 0, \quad r \in \Omega \quad (A.1)$$

with the boundary conditions of

$$B_\alpha(u, \frac{\partial u}{\partial n}) = 0, \quad r \in \Gamma \quad (A.2)$$

where  $D_\alpha$  is a general differential operator,  $B_\alpha$  is a boundary operator,  $f(r)$  is a known analytical function and  $\Gamma$  is the boundary of the domain  $\Omega$ . In general, the operator  $D_\alpha$  can be divided into a linear part  $L$  and a non-linear part  $N$ . Equation (A.1) can therefore be written as

$$L(u) + N(u) - f(r) = 0 \quad (A.3)$$

By the Homotopy technique, we construct a Homotopy  $H(v, p) : \Omega \times [0, 1] \rightarrow \mathbb{R}$  that satisfies

$$H(v, p) = (1-p)(L(v) - L(u_0)) \quad (A.4)$$

$$+ p[D_\alpha(v) - f(r)] = 0 \quad (A.4)$$

$$H(v, p) = L(v) - L(u_0) + pL(u_0) \quad (A.5)$$

where  $p \in [0, 1]$  is an embedding parameter, and  $u_0$  is an initial approximation of eqn. (A.1) that satisfies the boundary conditions. From the eqns. (A.4) and (A.5), we have

$$H(v, 0) = L(v) - L(u_0) = 0 \quad (A.6)$$

$$H(v, 1) = D_\alpha(v) - f(r) = 0 \quad (A.7)$$

When  $p=0$ , the eqns. (A.4) and (A.5) become linear equations. When  $p=1$ , they become non-linear equations. The process of changing  $p$  from zero to unity is that of  $L(v) - L(u_0) = 0$  to  $D_\alpha(v) - f(r) = 0$ .

We first use the embedding parameter  $p$  as a "small parameter" and assume that the solutions of



$$\begin{cases} \frac{\partial^\alpha u}{\partial t^\alpha} = \frac{\partial^2 u}{\partial x^2} + 2u \frac{\partial^\alpha u}{\partial x^\alpha} - \frac{\partial(uv)}{\partial x}, & (0 < \alpha \leq 1) \\ \frac{\partial^\beta v}{\partial t^\beta} = \frac{\partial^2 v}{\partial x^2} + 2v \frac{\partial^\beta v}{\partial x^\beta} - \frac{\partial(uv)}{\partial x}, & (0 < \beta \leq 1). \end{cases} \quad (1)$$

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ORIGINAL ARTICLE

Analytical approach to Fokker–Planck equation with space- and time-fractional derivatives by means of the homotopy perturbation method

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**KEYWORDS**  
Homotopy perturbation  
method;  
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equation;  
Caputo derivative

**Abstract** In this study, we present numerical solutions for the space- and time-fractional Fokker–Planck equation using the homotopy perturbation method (HPM). The fractional derivatives are described in the Caputo sense. The methods give an analytic solution in the form of a convergent series with easily computable components, requires no linearization or small perturbation. Some examples are given to show the validity and the great potential of the method. The homotopy perturbation method is very effective and convenient and overcome the difficulty of traditional methods. The numerical results show that the approaches are easy to implement and accurate when applied to space- and time-fractional Fokker–Planck equation. This study can be considered as a promising tool for solving many space-time fractional partial differential equations.

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1. Introduction

The Fokker–Planck equation arises in various fields in natural science, including solid-state physics, quantum optics, chemical physics, theoretical biology and circuit theory. The Fokker–Planck equation was first used by Fokker and Planck for instance, see Risken (1989), to describe the Brownian motion

of particles. A FPE describes the change of probability of a random function in space and time hence it is naturally used to describe solute transport. The general FPE for the motion of a particle in a medium with one space variable  $x$  at time  $t$  has the form (Risken, 1989)

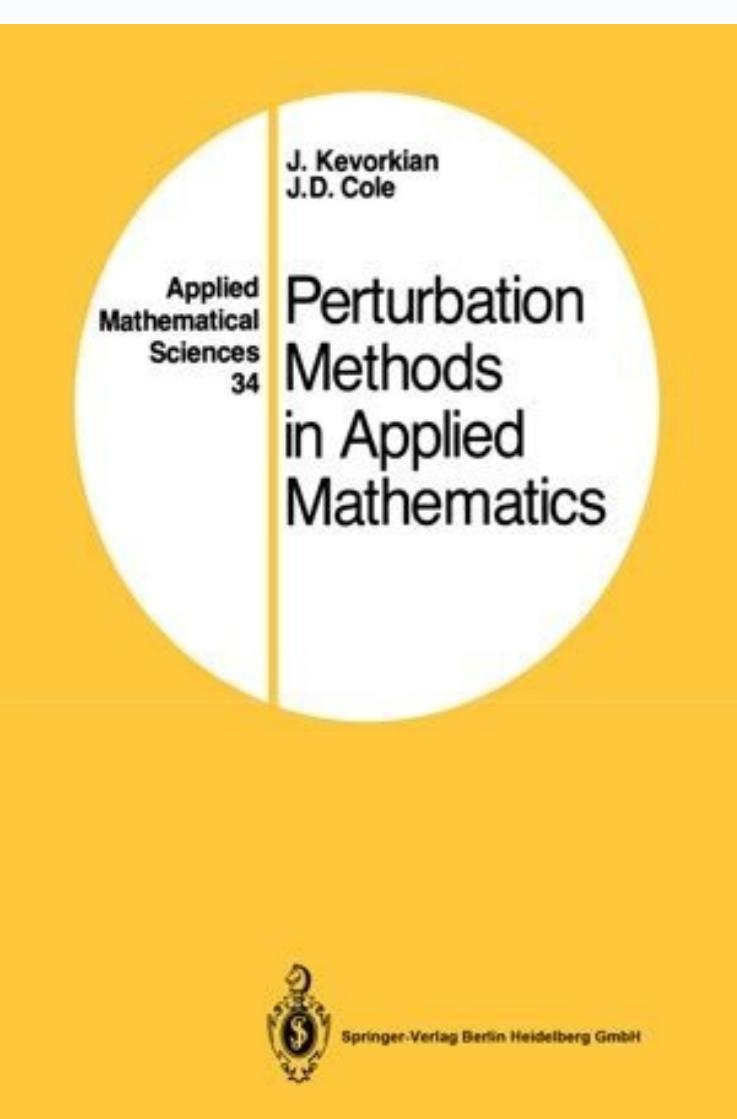
$$\frac{\partial u}{\partial t} = \left[ -\frac{\partial}{\partial x} f(x,t) + \frac{\partial^2}{\partial x^2} B(x,t) \right] u(x,t), \quad (1)$$

with the initial condition given by

$$u(x,0) = f(x), \quad x \in R, \quad (2)$$

where  $B(x,t) > 0$  is the diffusion coefficient and  $f(x,t)$  is the drift coefficient. The drift and diffusion coefficients may also depend on the velocity  $v$ , which is a vector and the scalar parameter  $\theta$ .

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Perturbation methods in applied mathematics.

In essence, a disturbance procedure consists of building the solution for a problem involving a small parameter  $b$ , either in the differential equation or in the limiting conditions or both, when the solution for the limiting case is known  $b = 0$ . . . . but its activity and behavior on this site made us think that oted is a bot, dca report is a revised and updated version, which includes a substantial part of new material, from J. A. erdeley, asymptotic expansions, dover publications, new york, 1956. series of electronic book information: applied mathematical sciences 34 years: 1,981 edition: 1: 1 pages: 560 pages: 568 language: English library: kolxo3 problem: 6897-278 4757-4213-8 dpi: 600 org file size: 4,307,881 extension: djvu tags: applied analysis toc: fractional matter . . . pages: 13 introduction . . . pages 1-16 limit processes . . . extensions applied to ordinary differential equations . . . pages 17-104 procedures of multiple variable expansions . . . pages 105-329 applications of partial differential equations . . . pages 330-480 expressions of fluid mechanics . . . pages 481-540 applications . . . pages 541-556 limit processes . . . pages 557-575https://doi.org/10.1016/0022-247X (82) 90139-1 Rights and content 5704 Access 755 Quotes Page 2 We will use conventional order symbols as a mathematical measure of the relative magnitude of several quantities. However, basic ideas are also applicable to integral equations, fundamental equations and even differences. Some of the most advanced ideas are reviewed as necessary; therefore, this book can serve as a text in an advanced undergraduate course or in a postgraduate course on the subject. Cole (Auth.) Djvu Download Embed This document was loaded by our user. The applied mathematician, who tries to understand or solve a physical problem, often uses a procedure of disturbance. This is a preview of the subscription content, access through your institution.

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